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# Numerical study of natural convection in square enclosure with a PCM wall

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***Abstract*—A model to predict the heat transfer in a differentially heated cavity which one of it's vertical plane wall is covered with a thin layer of phase change material (PCM) is proposed and discussed . The mathematical and numerical modeling will be first presented, then the boundary associated with the PCM wall is formulated in term of "Signorini problem". The numerical method proposed to solve this formulation is first validated by two test cases and a differentially heated cavity is studied.**

***Index Terms*—Natural convection, Differentially heated cavity, Heat transfer, Phase-change material, Signorini Problem.**

## I. Introduction

In the last decades the use of PCM is extensive in various fileds such as : buldings to improve their thermal inertia, thermal protection of electronic devices in order to limit the peak of temperature and increase battery life time, solar power plants, etc. The common goal of all these applications is the heat energy storage. Heat energy storage not only reduces the mismatch between the supply and the demand, but also improves the performance and reliability of energy systems, hence plays a crucial role in the future energy needs [1,2]. Phase change materials (PCMs) usually provide a large amount of heat, due to phase change during charging and discharging at a constant phase changing temperature [3-5]. Due to the rise of computer-aided modeling, several numerical methods for solving phase change problems have been developed. A phase change problem is more generally characterized by a moving melting front which splits the domain into a liquid and a solid phase. The overall phenomena is described by heat transfer equations and involves latent heat terms. The well known "Stefan problem" [6-8] which is well-studied since more than 100 years, illustrates this kind of phase change phenomena. Two types of numerical method have been developed to these problems. Firstly, the interface tracking of the melting front is based on a Lagrangian description of the interface. On the other hand, a derived formulation of the heat equation, namely enthalpy method [9-10], is based on an Eulerian description thanks to the introduction of the liquid fraction. In our work, we consider a PCM thin layer on the boundary of the fluid domain; we formulate this by means of "Signorini formulation" and we propose a new numerical method . The mathematical and numerical modeling is first presented and numerical results dealing with a differentially heated cavity which one of it's vertical plane wall is covered with a thin layer of PCM is next investigated to hilight the potential of this method.

## II. Mathematical and numerical modeling

### A. Geometry and governing equations

Let us consider a closed cavity containing an incompressible fluid (Fig-1). The right wall of this cavity is covered with a thin layer of PCM and is subjected to a pulsating heat flux. A constant temperature is imposed on the left wall of the cavity and the horizontal walls are adiabatics (See Fig-1). The boundary conditions prescribed to the cavity generate a natural convection flow and a heat transfer by conduction in the PCM layer. The equations that govern transfer by natural convection and conduction are linked at the vertical wall by conditions of temperature and heat flux continuities. The Obereck-Bousssinesq approximation is used to relate density to temperature variation, and to link in this way the temperature field "$\theta$" to the flow field [11,12]. The flow inside the cavity is governed by the Navier-Stokes equations which are given in their non-dimensional following form :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+(\frac{Pr}{Ra})^{1/2}(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial x}+(\frac{Pr}{Ra})^{1/2}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+\theta \tag{3}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=+(RaPr)^{-1/2}(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}) \tag{4}$$

In writing Equations (1-4) the dimensionless parameters are based on : the length scale $L_{ref}=H$; the reference velocity $u_{ref} = \frac{2g\beta H}{\alpha L q"}$, and $\Delta T_{ref} = q"H/k$. The dimensionless temperature being defined with $\theta = \frac{T-T_c}{\Delta T_{ref}}$, in this context Raileygh number is defined as $Ra = \frac{g\beta m H^3}{\alpha\nu\Delta T_{ref}}$ and Prandtl number as $Pr = \frac{\nu}{\alpha}$.

For the heat transfer by conduction in the PCM we assumed that its thickness is small in comparaison with $L_{ref}$. Its

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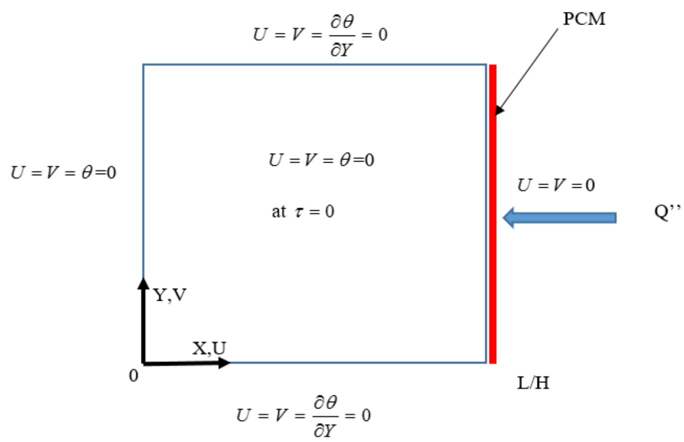



Figure 1. Two-dimensional enclosure heated with pulsating heat flux from the right side.

thermal inertia is neglected and the heat transfer occurs only in the normal direction. A thermal balance describes the heat transfer between the PCM and the fluid at the interface fluid-PCM as follow as :

$$(\mathbf{q_c} - \mathbf{j}).\mathbf{n} = \rho L_m \lambda' \; on \, \Gamma_{PCM} \quad (5)$$

with $\mathbf{q_c}$ the diffusive flux, $\mathbf{j}$ is the imposed heat flux, $\mathbf{n}$ is the outward normal, $L_m$ is the latent heat, $\rho$ is the density, and $\lambda'$ is the temporal derivative of the liquid fraction. So, the melting-solidifying model in the PCM is given by a system of inequalities :

$$\begin{cases} \lambda = 0 & if \; T \leq T_m \\ \lambda = 1 & if \; T > T_m \\ 0 \leq \lambda \leq 1 & if \; T = T_m \end{cases} \quad (6)$$

This model can be divided into the two following models

$$\begin{cases} (T_m - T)\lambda = 0 \\ T_m - T \geq 0 \\ \lambda \geq 0 \end{cases} \quad (7)$$

$$\begin{cases} (T - T_m)(1 - \lambda) = 0 \\ T - T_m \geq 0 \\ 1 - \lambda \geq 0 \end{cases} \quad (8)$$

Equations (7) describe the mushy-solid phase change and the equations (8) the mushy-liquid phase change. This boundary condition is similar to one of Signorini mechanical contact problem [13]. The literature, revealed a recent numerical method of the Signorini problem developed by Zhang and Zhu [13]. They derive an iterative method to solve the Signorini boundary condition over the Laplace equation. This method is retained in this work.

### B. Numerical methodology

Our formulation is different from that of the Signorini boundary condition, developed by Zhang and Zhu, by the lake of normal derivative term in our formulation. So first, we try to get closer to their formulation in order to use their iterative algorithm. Then, the temporal discretization of equation (5) expressed on equations (7) and (8) at each time step leads to the Signorini boundary condition on $\Gamma_{PCM}$ that is the PCM boundary of the solid part as follow as :

$$\begin{cases} (T_m - T^{n+1})(a - b\nabla T^{n+1}.n) = 0 \\ T_m - T^{n+1} \geq 0 \\ a \geq b\nabla T^{n+1}.n \end{cases} \quad on \, \Gamma_{MCP} \quad (9)$$

and the liquid part as follow as :

$$\begin{cases} (T_m - T^{n+1})(b\nabla T^{n+1}.n - (a-1)) = 0 \\ T^{n+1} - T_m \geq 0 \\ b\nabla T^{n+1}.n \geq a - 1 \end{cases} \quad on \, \Gamma_{MCP} \quad (10)$$

where $a = \lambda^n - \frac{\Delta t}{\rho L_m} j^{n+1}.n \quad and \quad b = \frac{\Delta t}{\rho L_m} k$.

These two parts are indeed complementary. The special feature of this formulation is that the two parts are valid in the mushy zone, which allows us to exchange between the two parts. In the next section, we present the iterative algorithm of resolution for the implicit scheme proposed.

### C. Algorithm

The method of solving the transfer equations is based on the projection method presented by Zhang and Zhu [13]. They propose to introduce a fixed point equation to solve nonlinear boundaries. We present the resolution to the solid part. For a constant c> 0, the equation of the solid phase is equivalent to the following fixed point equation:

$$T_m - T - [T_m - T - c(a - b\nabla T.n)]_+ = 0 \quad (11)$$

with $[x]_+ = max(x, 0)$. According to Zhang and Zhu, an iterative scheme is proposed as follows:

$$T^{(k+1)} = T_m - [T_m - T^{(k)} - c(a - b\nabla T^{(k+1)}.n)]_+ \quad (12)$$

Zhang et Zhu shown that for any initial value $T^0 > T_m$ and for any positive constant $c > 0$, $\{T^k\}$ converges to a unique solution.The iterative steps are :

- Step-1 : compute $S_d = \{x \in \Gamma_{MCP}, \quad T_m - T^k - c(a - b\nabla T^k.n) \leq 0\}$.
- Step-2 : Solve the heat equation with $T^{k+1} = T_m$ in $S_d$and $T^{k+1} + cb\nabla T^{k+1}.n = T^k + ca$ in $\Gamma_{MCP\S_d}$.
- Step-3 : If $\|T^{k+1} - T^k\| > \epsilon$, return to step 1, otherwise updates the liquid fraction $\lambda^{n+1}$ and advance a new time.

Regarding the liquid part of the boundary condition the calculation algorithm is the same, replacing $a$ par $a - 1$ in the algorithm.

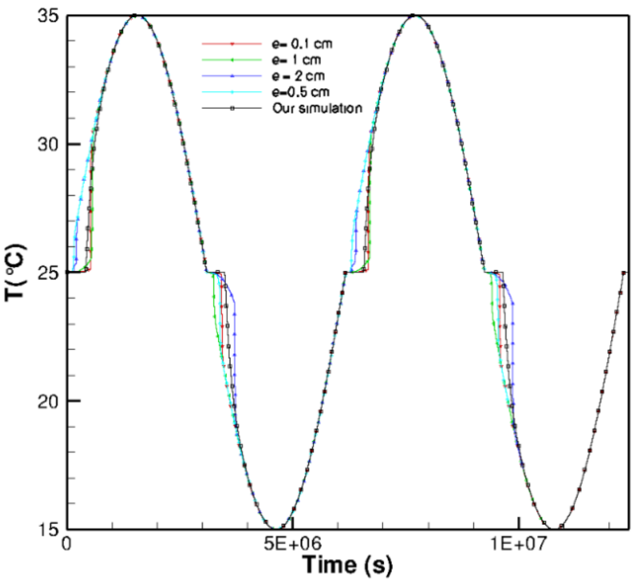



Figure 2. Average temperature profile at the interface rod-PCM

## III. Results

### A. Heat conduction in a rod with a PCM side

In order to validate the proposed numerical method, we consider a one dimensional case which allows us comparisons with standard methods. So, we consider the heat conduction problem in a solid bar which the left end is covered by a thin layer of PCM. The temperature at the left end of this bar is imposed. The heat transfer of the considered problem is composed of a common heat conduction in the bar and a phase change phenomena in the right part. These transfer equations are coupled by temperature and flux conditions at the interface rod-PCM. In the solid bar, heat transfer is governed by the unsteady heat conduction equation and in the PCM domain an enthalpy method is used (Voller method) [15]. To validate our model, the average interface temperature calculated with our numerical method and the multi-domain method [14] are compared. Indeed, the testing physical model is a rod of length 10 cm with a thermal diffusivity $\alpha = 2.61\,10^{-4}\,m^2/s$. Computations are carried out with four PCM thickness (e= 2 cm, 1 cm, 0.5 cm and 0.1 cm). It should be noted that the heat energy stored that is the product $L_v \times e = 55000\,Jm/kg$, is constant. It should be highlighted that our scheme is able to reproduce the time evolution of the average temperature at the interface rod-PCM. The average interface rod-PCM temperatures obtained by multidomain method and by our method are very close as the PCM thickness decreases (Fig-2). This results validate our method for a thin PCM layer thickness. We can also observe that the phase change occurs at melting temperature of the selected PCM whatever the phase change as melting or solidifying. In the next section, this new boundary condition is applied in a computational fluid dynamics case.

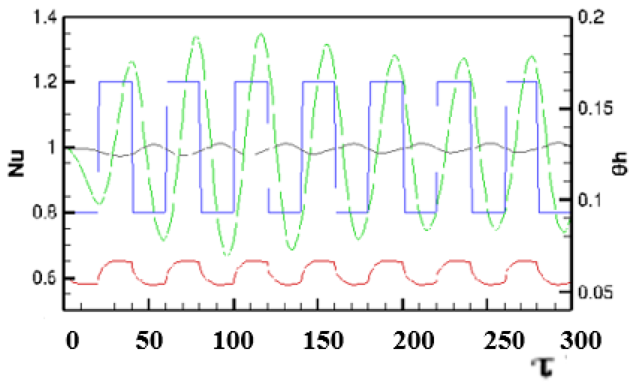
### B. Case of differentially heated cavity with a PCM layer on a vertical wall

The differentially heated cavity detailed in section (2.A) is considered. Regardless the PCM boundary conditions, the Navier-Stokes equations are discretized by means of second order finite difference defined on a straggered grid. Then, the semi-implicit scheme of Adams Bashforth Crank Nicholson is considered as temporal discretization. The velocity/pressure coupling is overcame by the projection method [16]. The method of solution for the linear systems resulting of these discretizations is based on Multigrid solver. Our code developped to solve Navier-Stokes equations is validated with Lage & Bejan results [17]. Computations have been performed for Rayleigh number $Ra = 10^8$, and Prandtl number $Pr = 0.7$ as well as a rectangular heat flux function $q"(t)$ which is applied on the right side with a dimensionless frequency $f = 0.025$. The figure 3 represents the evolution versus time of the average hot side temperature, the cold side Nusselt number and the Nusselt number at the vertical midplane at the established regime. It can be observed, that the evolution of these three physical parameters is periodic in time. The average hot side temperature and the Nusselt number at the vertical midplane are in phase, and they are in antiphase with the cold side Nusselt number. The amplitude for Nusselt number at the midplane exceeds the imposed rectangular flux. We can confirm that we obtain results similar to those of Lage and Bejan [17].

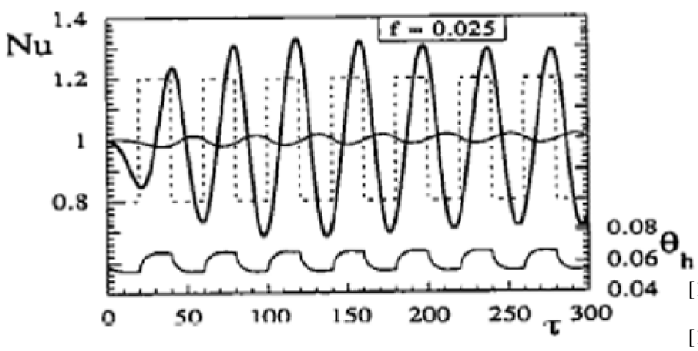
Now, we consider the numerical method dealing with the PCM heat transfer equation, which is coupled with Navier-Stokes equations. Then, we consider a thin PCM layer on the right wall of the square enclosure. The figure 4 presents the average hot side temperature on the left and Nusselt number at the midplane on the right for four non-dimensional melting temperature values (0.04, 0.05, 0.06, 0.08). The product $L_v \times e$ which correspond to the heat energy storage is set to $55000\ Jm/kg$. It can be see that a periodic behaviour is retrieved even if a PCM layer is added on the right side wall. The main difference between the wall with and without PCM layer is the observation of sinusoidale temperature variations versus time. For a melting temperature $T_m = 0.04$, the amplitude of the Nusselt number at the midplane is varying between 0.8 and 1.2 and for $T_m = 0.08$ this amplitude is in the range 0.75 to 1.3. The same behaviour is observed for the average hot side temperature. Indeed, the Nusselt number and the average hot side temperature depend on the melting temperature. As it decreases the difference between maximum and minimum values of the average temperature and the Nusselt number decreases. The melting temperature has no influence on the difference phase of the average hot side temperature and the Nusselt number. It can be concluded that the melting temperature has an impact on the amplitude of the average Nusselt number and the hot side temperature but does not affect the phase time.

## IV. Conclusion

In this work, a one dimensional heat conduction problem has been investigated in order to validate the thin PCM

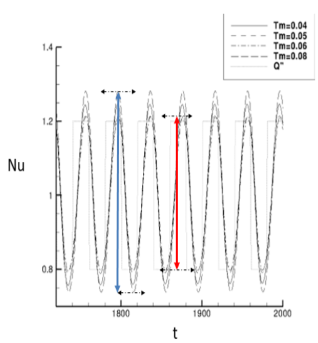



(a)

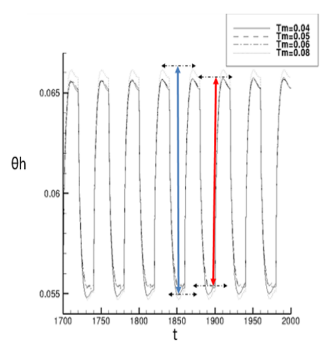



(b)

Figure 3. Our simulation : "Red" the average hot side temperature; "Green" the average Nusselt number; "Black" the cold side Nusselt number (a); Lage and Bejan Simulation (b)




(a)




(b)

Figure 4. The average Nusselt number (a) and the average hot side temperature (b)

boundary model proposed. This formulation has been applied to a differentially heated cavity with a vertical wall covered by a thin PCM layer thickness on the right side. It has been shown that the melting temperature reduces the difference between the maximum and minimum values of the average hot side temperature and the one of the Nusselt number.

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